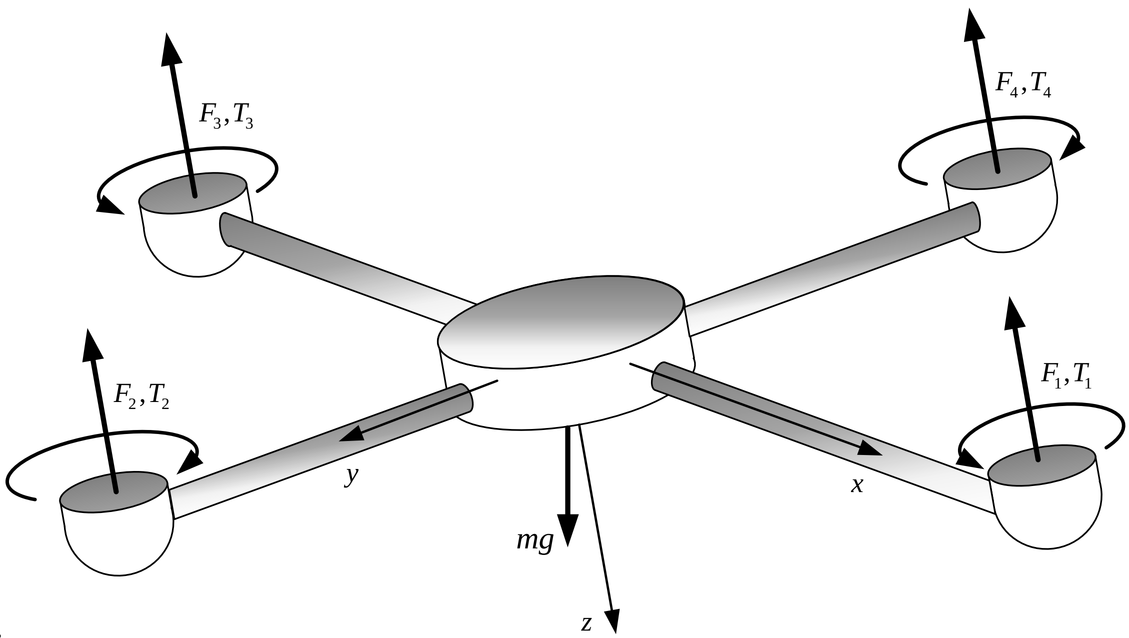
MEM 636 Fall Final Project, Fall 2019, Professor Kwatny

Due, Friday, December 6

Consider the quadcopter illustrated below



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The governing equations are:

Kinematics

$$\frac{d}{dt}\begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}=\begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix}x_s\\ y_s\\ z_s\end{bmatrix}=\left[\begin{array}{c|c|c}\cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi\\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi\\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta\end{array}\right]\begin{bmatrix}u\\ v\\ w\end{bmatrix}$$

Dynamics

$$\frac{d}{dt}\begin{bmatrix}p\\ q\\ r\end{bmatrix}=\begin{bmatrix}\frac{I_y-I_z}{I_x}qr\\ \frac{I_z-I_x}{I_y}pr\\ \frac{I_x-I_y}{I_z}pq\end{bmatrix}+\begin{bmatrix}\frac{\ell}{I_x}u_2\\ \frac{\ell}{I_y}u_3\\ \frac{1}{I_z}u_4\end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} rv - qw - g\sin\theta \\ pw - ru + g\cos\theta\sin\phi \\ qu - pv + g\cos\phi\cos\theta \end{bmatrix} + \frac{1}{m}\begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix}$$

Control forces:

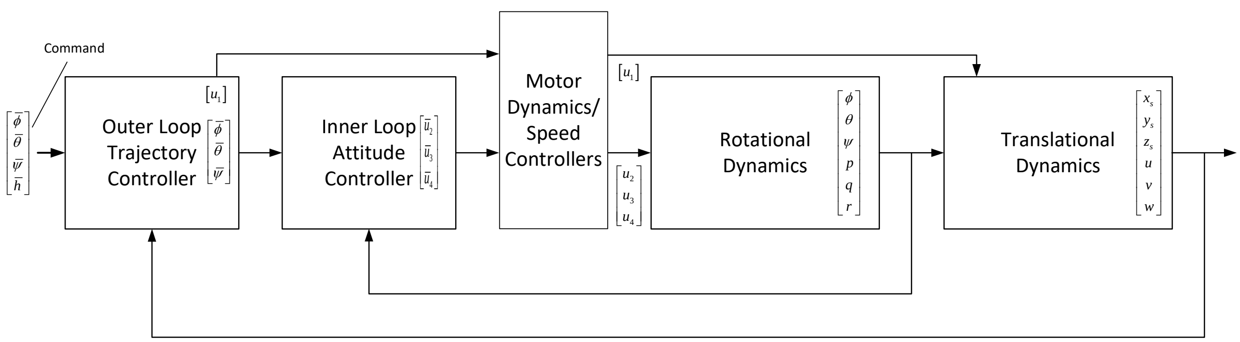
$$\text{roll:} \quad u_1 = \ell\left(F_4 - F_2\right) = \ell k\left(\omega_4^2 - \omega_2^2\right)$$

$$\text{pitch:} \quad u_2 = \ell\left(F_3 - F_1\right) = \ell k\left(\omega_3^2 - \omega_1^2\right)$$

$$\text{yaw:} \quad u_3 = -T_1 + T_2 - T_3 + T_4 = \kappa\left(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2\right)$$

$$\text{lift:} \quad u_4 = F_1 + F_2 + F_3 + F_4 = k\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right)$$

Notice that the translational dynamics are driven by the rotational dynamics but not vice-versa. So the control configuration is normally set up with a rotational inner loop and a translational outer loop as shown.



In this project, we will focus on the hover (altitude/rotational) control system. Altitude is $h = z_s$.

1. Consider the rotational tracking regulator problem with command model
   $\bar{\psi}(t) = w_1(t), \bar{\theta}(t) = w_2(t), \bar{\phi}(t) = w_3(t), \bar{h}(t) = w_4(t)$
   $\dot{w}_1 = 0$
   $\dot{w}_2 = 0$
   $\dot{w}_3 = 0$
   $\dot{w}_4 = w_5$
   $\dot{w}_5 = -w_4$

Parameters:

$$m \quad 0.468 \text{ kg}$$
$$\ell \quad 0.225 \text{ m}$$
$$I_x \quad 4.856\times10^{-3} \text{ kg m}^2$$
$$I_y \quad 4.856\times10^{-3} \text{ kg m}^2$$
$$I_z \quad 8.801\times10^{-3} \text{ kg m}^2$$
$$k \quad 2.980\times10^{-6}$$
$$\kappa \quad 1.140\times10^{-6}$$